

Exercise 37Calculate y' .

$$y = \sin\left(\tan\sqrt{1+x^3}\right)$$

SolutionCalculate y' by using the chain rule repeatedly.

$$\begin{aligned}y' &= \frac{d}{dx} \sin\left(\tan\sqrt{1+x^3}\right) \\&= \cos\left(\tan\sqrt{1+x^3}\right) \cdot \frac{d}{dx}\left(\tan\sqrt{1+x^3}\right) \\&= \cos\left(\tan\sqrt{1+x^3}\right) \cdot \left(\sec^2\sqrt{1+x^3}\right) \cdot \frac{d}{dx}\sqrt{1+x^3} \\&= \cos\left(\tan\sqrt{1+x^3}\right) \cdot \left(\sec^2\sqrt{1+x^3}\right) \cdot \frac{1}{2}(1+x^3)^{-1/2} \cdot \frac{d}{dx}(1+x^3) \\&= \cos\left(\tan\sqrt{1+x^3}\right) \cdot \left(\sec^2\sqrt{1+x^3}\right) \cdot \frac{1}{2}(1+x^3)^{-1/2} \cdot (3x^2) \\&= \cos\left(\tan\sqrt{1+x^3}\right) \left(\sec^2\sqrt{1+x^3}\right) \left(\frac{3x^2}{2\sqrt{1+x^3}}\right)\end{aligned}$$